N AND Δ RESONANCES

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I. Introduction

The excited states of the nucleon have been studied in a large number of formation and production experiments. The Breit-Wigner masses, pole positions, widths, and elasticities of the N and Δ resonances in the Baryon Summary Table come largely from partial-wave analyses of πN total, elastic, and charge-exchange scattering data. The most comprehensive analyses were carried out by the Karlsruhe-Helsinki (KH80) [1], Carnegie Mellon-Berkeley (CMB80) [2], and George Washington U (GWU) [3] groups. Partial-wave analyses have also been performed on much smaller πN reaction data sets to get $N\eta$, ΛK , and ΣK branching fractions. Other branching fractions come from analyses of $\pi N \to N\pi\pi$ data. A number of groups have undertaken multichannel analyses of these and associated photo-induced reactions (see Sec. VI).

In recent years, a large amount of data on photoproduction of many final states has been accumulated, and these data are beginning to make a significant impact on the properties of baryon resonances. A survey of data on photoproduction can be found in the proceedings of recent conferences [4] and workshops [5], and in a recent review [6].

II. Naming scheme for baryon resonances

In the past, when nearly all resonance information came from elastic πN scattering, it was common to label resonances with the incoming partial wave $L_{2I,2J}$, as in $\Delta(1232)P_{33}$ and $N(1680)F_{15}$. However, most recent information has come from γN experiments. Therefore, we have replaced $L_{2I,2J}$ with the spin-parity J^P of the state, as in $\Delta(1232)$ 3/2⁺ and N(1680) 5/2⁺. This applies to all baryons, including those such as the Ξ resonances and charm baryons that are not produced in formation experiments. Stable baryons $(N, \Lambda, \Sigma, \Xi, \Omega, \Lambda_c, \cdots)$ have no spin, parity, or mass attached.

III. Using the N and Δ listings

Tables 1 and 2 list all the N and Δ entries in the Baryon Listings and gives our evaluation of the status of each, the overall status,

Table 1. The status of the N resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

					Status as seen in —					
Particle J^P	overa	Status πN	γN	$N\eta$	$N\sigma$	ΛK	ΣK	$N\rho$	$\Delta \pi$	
$N = 1/2^{+}$	****									
$N(1440) 1/2^+$	****	****	****		***			*	***	
$N(1520) 3/2^-$	****	****	****	***				***	***	
$N(1535) 1/2^-$	****	****	****	****				**	*	
$N(1650) 1/2^-$	****	****	***	***		***	**	**	***	
$N(1675) 5/2^-$	****	****	***	*		*		*	***	
$N(1680) 5/2^+$	****	****	****	*	**			***	***	
$N(1700) 3/2^-$	***	***	**	*		*	*	*	***	
$N(1710) 1/2^+$	***	***	***	***		***	**	*	**	
$N(1720) 3/2^+$	****	****	***	***		**	**	**	*	
$N(1860)5/2^+$	**	**						*	*	
$N(1875) 3/2^-$	***	*	***			***	**		***	
$N(1880) 1/2^+$	**	*	*		**	*				
$N(1895) 1/2^-$	**	*	**	**		**	*			
$N(1900) 3/2^+$	***	**	***	**		***	**	*	**	
$N(1990) 7/2^+$	**	**	**				*			
$N(2000)5/2^+$	**	*	**	**		**	*	**		
$N(2060) 5/2^-$	**	**	**	*			**			
$N(2150) 3/2^-$	**	**	**			**			**	
$N(2190) 7/2^-$	****	****	***			**		*		
$N(2220) 9/2^+$	****	****								
$N(2250) 9/2^-$	****	****								
$N(2600) 11/2^-$	***	***								
$N(2700) 13/2^+$	**	**								

^{****} Existence is certain, and properties are at least fairly well explored.

^{***} Existence is very likely but further confirmation of quantum numbers and branching fractions is required.

^{**} Evidence of existence is only fair.

^{*} Evidence of existence is poor.

Table 2. The status of the Δ resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

				Status as seen in —					
Particle J^P	overa	Status πN		$N\eta$	$N\sigma$	ΛK	ΣK	$N\rho$	$\Delta \pi$
$\Delta(1232) \ 3/2^+$	****	****	****	F					
$\Delta(1600) \ 3/2^{+}$	***	***	***	О				*	***
$\Delta(1620) \; 1/2^-$	****	****	***	\mathbf{r}				***	***
$\Delta(1700) \ 3/2^-$	****	****			b			**	***
$\Delta(1750) \ 1/2^{+}$	*	*			i				
$\Delta(1900) \ 1/2^-$	**	**	**		d		**	**	**
$\Delta(1905) 5/2^+$	****	****	****		d		***	**	**
$\Delta(1910) 1/2^+$	****	****	**			e	*	*	**
$\Delta(1920) \ 3/2^+$	***	***	**			n	***		**
$\Delta(1930) \ 5/2^-$	***	***							
$\Delta(1940) \ 3/2^-$	**	*	**	F			(seen	in	$\Delta \eta)$
$\Delta(1950) \ 7/2^+$	****	****	****	О			***	*	***
$\Delta(2000) 5/2^+$	**			r					**
$\Delta(2150) \; 1/2^-$	*	*			b				
$\Delta(2200) \ 7/2^-$	*	*			i				
$\Delta(2300) 9/2^+$	**	**			d				
$\Delta(2350) \ 5/2^-$	*	*			Ċ	l			
$\Delta(2390) \ 7/2^+$	*	*				e			
$\Delta(2400) 9/2^-$	**	**				n			
$\Delta(2420) \ 11/2^{+}$	****	****	*						
$\Delta(2750) \ 13/2^-$		**							
$\Delta(2950) \ 15/2^{+}$		**							

^{****} Existence is certain, and properties are at least fairly well explored.

^{***} Existence is very likely but further confirmation of quantum numbers and branching fractions is required.

^{**} Evidence of existence is only fair.

^{*} Evidence of existence is poor.

the status from $\pi N \to N\pi$ scattering data and from photoproduction experiments, and the status channel by channel. Only the established resonances (overall status 3 or 4 stars) are promoted to the Baryon Summary Table. We have omitted from the Listings information from old analyses, prior to KH80 and CMB80 which can be found in earlier editions. A rather complete survey of older results was given in our 1982 edition [7].

The star rating assigned to a resonance depends on the data base and the analysis. As a rule, we award an overall status *** or **** only to those resonances which are confirmed by independent analyses and which are derived from analyses based on complete information, i.e. for analyses based on three observables in πN scattering or eight properly chosen observables in photoproduction. Use of dispersion relations (as in the KH80, CMB80, and GWU analyses) may lift these requirements. Three and four-star resonances should be observed in a channel in which they dominate the background. For resonances with high inelasticity, the smallest background is expected when single-energy partial-wave amplitudes of appropriate inelastic channels are constructed. Weak signals or signals emerging in analyses with incomplete experimental information are given ** or * status. A new category, called Further States lists observations of resonances which do not yet deserve a one-star status. We do not consider new results without proper error evaluation. Finally, we caution against the association of new states with those previously rated as one- and two-star states, found in the KH80 and CMB80 analyses, as these fits are in significant disagreement with more recent spin-rotation parameter measurements. See our 2010 edition for more details [8].

In the Data Listings, we give first the position of the pole of a resonance and its elastic pole residue, then the Breit-Wigner parameters. We warn the reader that Breit-Wigner parameters depend on the formalism used; in particular the width depends critically on the treatment of the phase volume of multiparticle decays. For the first time, we give residues and phases of hadronic transition amplitudes and helicity amplitudes. The Breit-Wigner parameters follow, including branching ratios, since these parameters are often used in comparison with models.

IV. Properties of resonances

Resonances are defined by poles of the scattering amplitude in the complex energy (\sqrt{s}) plane [9]. In contrast to other quantities related to resonance phenomena, such as the Breit-Wigner mass or the K-matrix pole, a pole of the scattering amplitude does not depend on the gauge chosen, and production and decay properties factorize. It is the pole position which should be compared to eigenvalues of the Hamiltonian of full QCD. Examining the Listings, one finds a much larger spread in Breit-Wigner parameters compared to pole parameters.

In scattering theory, the amplitude for the scattering process leading from the initial state a to the final state b is given by the S matrix, which can be decomposed as follows:

$$S_{ab} = (I + 2i\rho T)_{ab} . \tag{1}$$

Here I_{ab} is the identity operator, and T_{ab} describes the transition from the initial state to the final state (e.g. πN , to ΣK); and ρ_{ab} is a matrix of phase-space elements. The transition amplitude T contains poles due to resonances and background terms. The specification of charge states requires additional Clebsch-Gordan coefficients. Above the threshold for inelastic reactions, a resonance is associated with a cluster of poles in different Riemann sheets. The pole closest to the real axis has the strongest impact on the data. It is situated on the second Riemann sheet starting at the highest threshold below the pole position. If the threshold is close to the pole position, poles in other sheets may have an important impact as well. Other complications may occur: Broad resonances are difficult to disentangle from background amplitudes, e.g. due to lefthand cuts originating from meson and baryon exchange forces. A two-particle subsystem generates a square-root singularity at its threshold; poles in a two-body subsystem, e.g. the ρ meson in the $\pi\pi$ system, lead to branch points in the complex energy plane. Neglecting some of these aspects leads to a model dependence of the pole position. These uncertainties increase with the particle width.

Several particle properties are related to poles. First, poles exist on multiple Riemann sheets. In the Listings, we give for each resonance the position of the most relevant pole. Within a model, the poles of the scattering amplitude can be found by analytic continuation of the amplitude; by scanning the energy plane for extreme values; by the time-delay or speed-plot technique [10]; or by calculating the trace of T matrices [11]. The real part of the pole position defines the particle mass $\text{Re}(s_{\text{pole}}) = m_{\text{pole}}$; the imaginary part its half width $-\text{Im}(s_{\text{pole}}) = \Gamma_{\text{pole}}/2$.

Further particle properties are derived as residues of transition amplitudes. These are calculated through a contour integral of the amplitude T_{ab} around the pole position in the energy plane:

$$Res(a \to b) = \oint \frac{d\sqrt{s}}{2\pi i} \sqrt{\rho_a} T_{ab}(s) \sqrt{\rho_b}$$

$$= \frac{1}{2m_{\text{pole}}} \sqrt{\rho_a(m_{\text{pole}}^2)} g_a g_b \sqrt{\rho_b(m_{\text{pole}}^2)}, \qquad (2)$$

For elastic scattering, this gives the elastic residue:

$$Res(\pi N \to N\pi) = \frac{1}{2m_{\text{pole}}} \rho_a(m_{\text{pole}}^2) g_a^2.$$
 (3)

The factor ρ contains the orbital-angular-momentum barrier, the nucleon propagator, and the two-body phase space according to

$$\rho_{\pi N}^{L} = \frac{|\vec{k}|^{2L}}{F(L, r^2, k^2)} \cdot \frac{m_N + k_{N0}}{2m_N} \cdot \frac{1}{16\pi} \frac{2|\vec{k}_a|}{\sqrt{s}}, \tag{4}$$

where k_{N0} is the c.m. nucleon energy; $F(L, r^2, k^2)$ is a form factor, e.g. the Blatt-Weisskopf form, which depends on the linear (k) and the orbital-angular (L) momenta and an interaction radius r. The factor $1/(2m_{\text{pole}})$ in Eq. (3) comes from $d\sqrt{s} = ds/(2\sqrt{s})$. g_a is the coupling of the resonance to the final state a. Branching ratios of a pole can be defined by

$$Br_{\text{pole}}(\text{channel } b) = \frac{[Res(\pi N \to b)]^2}{Res(\pi N \to N\pi) \cdot (-\text{Im}(s_{\text{pole}}))}.$$
 (5)

This information is, however, not given in the literature.

Within models, background amplitudes can be parameterized using an effective Lagrangian approach, in dynamical coupled channel approaches, or by low-order polynomial functions. Resonances are then added, often in the form of Breit-Wigner amplitudes. In the Listings, particle properties related to fits to data using Breit-Wigner amplitudes are given as well. These are the Breit-Wigner mass and width, the partial decay widths, and the branching ratios. It should be noted that Breit-Wigner parameters depend on the background parameterization.

The multichannel relativistic Breit-Wigner amplitude is given by

$$A_{ab} = \frac{-g_a g_b}{s - m_{\text{BW}}^2 + i \sum_{a} g_a^2 \rho_a(s)}$$
 (6)

where $m_{\rm BW}$ is called Breit-Wigner mass. In case of two channels, Eq. (6) is known as the Flatté formula. The imaginary part of its denominator at $s=M^2$ is defined as $M_{BW}\Gamma$, and the phase space ρ can be approximated by \sqrt{s} . This reduces expression (6) to the elastic amplitude (39.57) in section 39.

The partial width at the resonance position is defined as

$$m_{BW}\Gamma_a = g_a^2 \rho_a(m_{BW}^2)$$

$$BR_a = \Gamma_a/\Gamma$$
(7)

is the branching ratio for the decay of a resonance into channel a. Γ_a vanishes by definition for decay modes with thresholds above the Breit-Wigner mass. $\sum_a BR_a = 1$ follows from the definition. Unobserved decay modes lead to the inequality $\sum_a BR_a \leq 1$. In case of broad resonances definition (7) may be counter-intuitive. Branching ratios can also be defined as

$$BR' = \int_{\text{threshold}}^{\infty} \frac{ds}{\pi} \frac{g_a^2 \rho(s)}{(m_{\text{BW}}^2 - s)^2 + (\sum_a g_a^2 \rho_a(s))^2}.$$
 (8)

These branching ratios include decays of resonances into channels with thresholds above their nominal masses. The relation $\sum_a BR'_a = 1$ is needed for normalization.

V. Electromagnetic interactions

A new approach to the nucleon excitation spectrum is provided by dedicated facilities at the Universities of Bonn and

Mainz, and at the national laboratories Jefferson Lab in the US and SPring-8 in Japan. High-precision cross sections and polarization observables in photoproduction of pseudoscalar mesons provide a data set that is nearly a "complete experiment," one that fully constrains the four complex amplitudes describing the spin-structure of the reaction. A large number of photoproduction reactions has been studied.

In photoproduction, the spins of photon and nucleon can be parallel or anti-parallel, and there are spin-flip and non-flip transitions. Four independent amplitudes can be defined using the photon polarization and the hadronic current [12]. The amplitudes can be expanded into a series of electric and magnetic multipoles. In general, two amplitudes, one electric and one magnetic, contribute to one J^P combination. For a given resonance, these two amplitudes are related to the helicity amplitudes $A_{1/2}$ and $A_{3/2}$. The final state may have isospin I = 1/2 or I = 3/2.

If a Breit-Wigner parametrization is used, the $N\gamma$ partial width, Γ_{γ} , is given in terms of the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ by

$$\Gamma_{\gamma} = \frac{k^2}{\pi} \frac{2m_N}{(2J+1)m_{BW}} \left(|A_{1/2}|^2 + |A_{3/2}|^2 \right) . \tag{9}$$

Here m_N and m_{BW} are the nucleon and resonance masses, J is the resonance spin, and k is the photon c.m. decay momentum. Most earlier analyses have quoted these real quantities.

Other more recent studies have quoted related complex quantities, evaluated at the T-matrix pole. Here, the helicity-dependent amplitudes, $a^{1/2}$ and $a^{3/2}$, for photoproduction of the final state b, are

$$Res\left((\gamma N)^h \to b\right) = \frac{a^h g_b}{2m_{\text{pole}}} \sqrt{\rho_b(m_{\text{pole}}^2)}.$$
 (10)

They are renormalized by $|a^h|^2 = \frac{k^2}{\pi} \frac{2m_N}{(2J+1)} |A^h|^2$ to yield the helicity amplitudes, $A^{1/2}$ and $A^{3/2}$, of a resonance:

$$\Gamma_{\gamma} = \frac{1}{m_{\text{pole}}} \left(|a^{1/2}|^2 + |a^{3/2}|^2 \right) = \frac{k^2}{\pi} \frac{2m_N}{(2J+1)m_{\text{pole}}} \left(|A_{1/2}|^2 + |A_{3/2}|^2 \right). \tag{11}$$

The amplitudes $A_{1/2}$ and $A_{3/2}$, the elastic residues, and the residues of the transition amplitudes are complex numbers. The relation $(g_{N\pi})^2 = m_{\text{pole}}\Gamma_{N\pi}$ defines $g_{N\pi}$ up to a sign. Due to Eq. (10), the phase of the helicity amplitude depends on this definition. We define the phase of $g_{N\pi}$ clockwise.

The determination of eight real numbers from four complex amplitudes (with one overall phase undetermined) requires at least seven independent data points. At least one further measurement is required to resolve discrete ambiguities that result from the fact that data are proportional to squared amplitudes. Photon beams and nucleon targets can be polarized (with linear or circular polarization P_{\perp} , P_{\odot} and \vec{T} , respectively); the recoil polarization of the outgoing nucleon \vec{R} can be measured. The experiments can be divided into three classes: those with polarized photons and a polarized target (BT): and those measuring the baryon recoil polarization and using either polarized photons (BR) or a polarized target (TR). Different sign conventions are used in the literature, as summarized in [14].

A large number of polarization observables has been determined which constrain energy dependent partial wave solutions. One of the best studied reactions is $\gamma p \to \Lambda K^+$. Published data include differential cross sections, the beam asymmetry Σ , the target asymmetry T, the recoil polarization P, and the BR double-polarization variables C_x, C_z, O_x , and O_z . For $\gamma p \to p \pi^0$, $\gamma p \to n \pi^+$, and $\gamma p \to p \eta$, differential cross sections and beam asymmetry have been published; BT data for E, F, G, and H have been presented at conferences [15].

Electro-production of mesons provides information on the internal structure of resonances. The helicity amplitudes become functions of the momentum transfer, and a third amplitude, $S_{1/2}$ contributes to the process. Recent experimental achievements and their interpretation are reviewed by I.G. Aznauryan and V.D. Burkert [16].

VI. Partial wave analyses

Several PWA groups are now actively involved in the analysis of the new data. Of the three "classical" analysis groups at KH, CMB, and GWU, only the GWU group is still active. This group maintains a nearly complete database covering reactions

from πN and KN elastic scattering to $\gamma N \to N\pi$, $N\eta$, and $N\eta'$. It is presently the only group determining energy independent πN elastic amplitudes from scattering data. Given the high-precision of photoproduction data already collected and to be taken in the near future, we estimate that an improved spectrum of N and Δ resonances should become available in the forthcoming years.

Energy-dependent fits are performed by various groups. Ideally, the Bethe-Salpeter equation should be solved to describe the data. For practical reasons, approximations have to be made. We mention here: (1) The Mainz unitary isobar model focusses on the correct treatment of the low-energy domain; resonances are added to the unitary amplitude as sums of Breit-Wigner amplitudes. (2) Multichannel analyses using Kmatrix parameterizations derive background terms from a chiral Lagrangian (Giessen, KVI), or from phenomenology (Bonn-Gatchina). (3) Several groups (Bonn-Jülich, Argonne-Osaka, Valencia) use dynamical reaction models. Several other groups have made important contributions. So far, only the Bonn-Gatchina group, exploiting a very large data set on pion- and photo-induced reactions, has reported systematic searches for new baryon resonances in all relevant partial waves. A summary of their results can be found in [17].

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 $N(1860)5/2^+$

NEW Status: **

The former $N(2000)5/2^+$ is replaced by two entries, $N(1860)5/2^+$ and $N(2050)5/2^+$. Older results have been retained because there is little information at all about this possible state.

 $N(1875)3/2^-$

NEW Status: **

The new evidence for two resonances from Anisovich 2012 supports the original claim of Cutkosky 80. The former $N(2090)3/2^-$ is now replaced by two entries, $N(1875)3/2^-$ and $N(2110)3/2^-$.

Most of the results published before 1975 are now obsolete and have been omitted. They may be found in our 1982 edition, Physics Letters 111B 1 (1982). Some further obsolete results were last included in our 2006 edition, Journal of Physics, G 33 1 (2006).

 $N(1880)1/2^{+}$

NEW Status: **

Evidence for a new resonance is reported by Anisovich 2012, supporting the original claim by Manley 92.

 $N(1895)1/2^-$

NEW Status: **

Evidence for a new resonance is reported by Anisovich 2012, supporting the original claim by Hoehler 79 and Manley 92. The observation of Cutkosky 80 is moved to the new category *Further States*.

A few early results that are now obsolete have been omitted.

 $N(1900)3/2^{+}$

Status: **

Evidence for this resonance was reported by Manley 92 and Nikonov 08. Anisovich 2012 confirm the existence and report that the region could be split into two resonances.

 $N(2050)5/2^+$

NEW Status: **

The former $N(2000)5/2^+$ is replaced by two entries, $N(1860)5/2^+$ and $N(2050)5/2^+$. See $N(1860)5/2^+$.

 $N(2060)5/2^+$

NEW Status: **

The mass of the former $N(2200)5/2^-$ was not well determined. It is found by Anisovich 12 to be considerably lower and is now called $N(2060)5/2^-$.

 $N(2100)1/2^-$

omitted from Tables

There is only one observation of a structure above 2000 MeV, by Cutkosky 80. It is moved to the new category *Further States*.

 $N(2110)3/2^{-}$

NEW Status: **

The former $N(2080)3/2^-$ is now replaced by two entries, $N(1875)3/2^-$ and $N(2110)3/2^-$. See $N(2110)3/2^-$.

 $\Delta(1750)1/2^{+}$

omitted from Tables

This state has not been observed by Hoehler 79, Cutkosky 80, Arndt 06, nor by Anisovich 12. It is moved to the new category Further States.

Table 3. Further states. This table lists observations of poorly established states which do not yet (or no longer) deserve a \ast rating.

Mass	width	I	J^P	seen in	ref.
2180 ± 80 1744 ± 36		,	$\frac{1/2^{-}}{1/2^{+}}$	$\pi N \to N\pi$ $\pi N \to N\pi \& N\pi$	Cutkosky 80 $\pi\pi$ Manley 92